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Water quality control by bank placement based on optimal control and finite element method

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SUMMARY

This paper presents a method for quality control by bank placement based on an optimal control theory and the finite element method. The shallow water equation is employed for the analysis of the flow condition and the advection-diffusion equation is used for the analysis of pollutant concentration. The optimal control theory is utilized to obtain a control value for the objective state value. The shear-slip mesh update method which is suitable for the rotational problem of body is employed. To solve the optimization problem, the time domain decomposition method is applied as a technique of storage requirements reduction. The Sakawa–Shindo method is employed as a minimization technique. The Crank–Nicolson method is applied to the temporal discretization. A method for optimal control of bank placement has been presented. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: optimal control theory; finite element method; time domain decomposition method; Sakawa–Shindo method; shear-slip mesh update method

1. INTRODUCTION

There are many closed bays having serious water quality problem caused by effluents from factories and homes in Japan. To solve this type of pollution problem, the government has planned several projects. The first plan is to arrange drains. The second plan is to purify the polluted water area by utilizing the process of biodegradation. This plan is to attempt purification to create the sandy beach, sedimentation dredge and so on. These plans may not be applicable to purify the polluted water area for short time durations. The third plan is to promote the exchange of sea-water. This plan is for reducing the pollutant concentration by the injection of clear water. This plan is very efficient to purify the polluted water area for a short time duration, but it costs much to maintain the control device.

In this research, the method for controlling the pollutant concentration by the bank is studied. In the case of this plan, the location for construction of the bank to control the pollutant concentration is unknown. To decide on the location of the bank the optimal control

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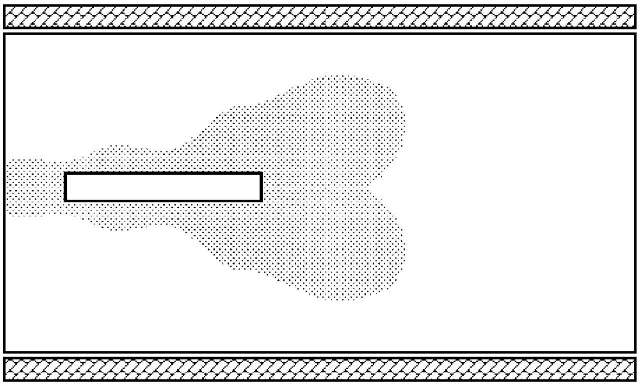
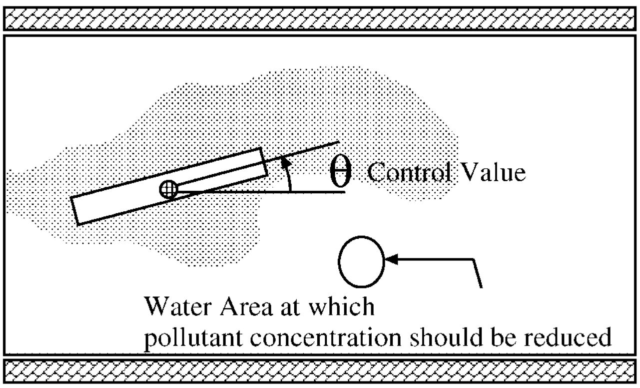

(a)



(b)

Figure 1. Image of this research: (a) non control; (b) optimal control.

theory is employed. In this research, to decide on the location of the bank, the optimal angle of rotation of the bank from the initial location is employed. Thus the purpose of this research is to confirm the effect of this plan which is the construction of a bank for controlling the pollutant concentration and to decide the optimal angle of rotation of bank for controlling the pollutant concentration at the objective point using the optimal control theory and finite element method. The angle of rotation of bank is employed as the control value. The image of this research is illustrated in Figure 1.

For resolving the rotational problem of the body, the remeshing technique is required. For the remeshing technique, the shear-slip mesh update method is used in this paper. This method is very efficient for solving the rotational problem of the body.

To solve the optimization problem, especially for large scale computation, computational storage is needed for which every solution of the state equation is required to calculate the adjoint equation. Therefore the technique of storage saving should be introduced. To do this, the time domain decomposition method is employed as a technique for reducing storage requirements.

In this research, as case studies, two numerical simulations are carried out. The first example is the verification of the present method using the problem of control of pollution in the rectangular channel. The second example is the problem of control of pollution at the Onjuku Coast located in Chiba prefecture in Japan.

2. STATE EQUATION

2.1. State equation

The shallow water equation is employed to calculate the flow, and the advection-diffusion equation is employed to calculate the distribution of pollutant concentration. The shallow water equation is written as

$$\dot{u}_i + u_j u_{i,j} + g\eta_{,i} - v(u_{i,j} + u_{j,i})_{,j} + f u_i = 0 \tag{1}$$

$$\dot{\eta} + \{(h+\eta)u_i\}_{,i} = 0 \tag{2}$$

where u_i denotes water velocity, η represents the water elevation, g is the gravity acceleration and h is water depth, respectively. Kinematic eddy viscosity is expressed by v, which is expressed as

$$v = \frac{\kappa_l}{6} u_* h$$

where κ_l is a Kalman constant and u_* is the velocity of friction which is given as

$$u_* = \frac{gn^2 u_k^2}{h^{1/3}}$$

and f, the bottom of friction term, can be denoted as

$$f = \frac{gn^2}{(h+\eta)^{4/3}} \sqrt{u_k u_k}$$

where n is the Manning coefficient of roughness. The boundary condition of the shallow water equation is given as follows:

$$u_i = \hat{u}_i \quad \text{on } \Gamma_d \tag{3}$$

$$\eta = \hat{\eta} \quad \text{on } \Gamma_d \tag{4}$$

$$u_n = u_i n_i = \hat{u}_n \quad \text{on } \Gamma_n \tag{5}$$

The advection-diffusion equation is represented as

$$\dot{c} + u_i c_{,i} - \kappa c_{,ii} = 0 \tag{6}$$

where c is the pollutant concentration and κ the diffusion coefficient. The boundary condition of the advection-diffusion equation is given as

$$c = \hat{c} \quad \text{on } \Gamma_d \tag{7}$$

$$b = \kappa c_{,i} n_i = \hat{b} \quad \text{on } \Gamma_n \tag{8}$$

where n_i denotes the direction cosine of the unit outward normal of the boundary and $\hat{b}$ expresses the concentration flux on the boundary. The overhat ˆ expresses the given value on the boundary.

The initial condition is given as

$$u_i = \hat{u}_{i0} \quad \text{in } \Omega \text{ at } t = t_0 \tag{9}$$

$$c = \hat{c}_0 \quad \text{in } \Omega \text{ at } t = t_0 \tag{10}$$

2.2. Finite element equation

Multiplying weighting function w by both sides of Equations (1), (2) and (3) and integrating over the domain Ω, the weighted residual equations can be obtained as

$$\begin{aligned} &\int_\Omega w\dot{u}_i \,\mathrm{d}\Omega + \bar{u}_j \int_\Omega w u_{i,j} \,\mathrm{d}\Omega + g \int_\Omega \omega \eta_{,i} \,\mathrm{d}\Omega \\ &\quad - v \left(\int_\Omega w u_{i,jj} \,\mathrm{d}\Omega + \int_\Omega w u_{j,ij} \,\mathrm{d}\Omega \right) + f \int_\Omega w u_i \,\mathrm{d}\Omega = 0 \end{aligned} \tag{11}$$

$$\int_\Omega w\dot{\eta} \,\mathrm{d}\Omega + \bar{u}_i \int_\Omega wh \,\mathrm{d}\Omega + \bar{u}_i \int_\Omega w\eta_i \,\mathrm{d}\Omega + \overline{(h+\eta)} \int_\Omega w u_{i,i} \,\mathrm{d}\Omega = 0 \tag{12}$$

$$\int_\Omega w\dot{c} \,\mathrm{d}\Omega + \bar{u}_i \int_\Omega w c_{,i} \,\mathrm{d}\Omega - \kappa \int_\Omega w c_{,ii} \,\mathrm{d}\Omega = 0 \tag{13}$$

The weighted residual equations can be transformed into

$$\begin{aligned} &\int_\Omega w\dot{u}_i \,\mathrm{d}\Omega + \bar{u}_j \int_\Omega w u_{i,j} \,\mathrm{d}\Omega + g \int_\Omega w \eta_{,i} \,\mathrm{d}\Omega \\ &\quad + v \left(\int_\Omega w_{,j} u_{i,j} \,\mathrm{d}\Omega + \int_\Omega w_{,j} u_{j,i} \,\mathrm{d}\Omega \right) + f \int_\Omega w u_i \,\mathrm{d}\Omega = 0 \end{aligned} \tag{14}$$

$$\int_\Omega w\dot{\eta} \,\mathrm{d}\Omega + \bar{u}_i \int_\Omega wh \,\mathrm{d}\Omega + \bar{u}_i \int_\Omega w\eta_i \,\mathrm{d}\Omega + \overline{(h+\eta)} \int_\Omega w u_{i,i} \,\mathrm{d}\Omega = 0 \tag{15}$$

$$\int_\Omega w\dot{c} \,\mathrm{d}\Omega + \bar{u}_i \int_\Omega w c_{,i} \,\mathrm{d}\Omega + \kappa \int_\Omega w_{,i} c_{,i} \,\mathrm{d}\Omega = 0 \tag{16}$$

Interpolating the weighted residual equations using the bubble function element [1–4], the finite element equation can be obtained as

$$M\dot{u}_i + \bar{u}_j S_j u_i + g S_i \eta + \nu(H_{jj} u_i + H_{ji} u_j) + f M u_i = 0 \tag{17}$$

$$M\dot{\eta} + \bar{u}_i S_i h + \bar{u}_i S_i \eta + \overline{(h+\eta)} S_i u_i = 0 \tag{18}$$

$$M\dot{c} + \bar{u}_i S_i c + \kappa H_{ii} c = 0 \tag{19}$$

where the coefficient matrices of the finite element equation can be derived as follows:

$$M = \int_\Omega \Phi_\alpha \Phi_\beta \, \mathrm{d}\Omega, \quad S_i = \int_\Omega \Phi_\alpha \Phi_{\beta,i} \, \mathrm{d}\Omega, \quad S_j = \int_\Omega \Phi_\alpha \Phi_{\beta,j} \, \mathrm{d}\Omega$$

$$H_{ii} = \int_\Omega \Phi_{\alpha,i} \Phi_{\beta,i} \, \mathrm{d}\Omega, \quad H_{ji} = \int_\Omega \Phi_{\alpha,j} \Phi_{\beta,i} \, \mathrm{d}\Omega, \quad H_{jj} = \int_\Omega \Phi_{\alpha,j} \Phi_{\beta,j} \, \mathrm{d}\Omega$$

2.3. Temporal discretization

For the temporal discretization of the finite element equations, the Crank–Nicolson method is applied. The finite element equations discretized in the temporal direction can be obtained as

$$M\,\frac{u_i^{n+1} - u_i^n}{\Delta t} + \bar{u}_j S_j u_i^{n+1/2} + g S_i \eta^{n+1/2} + \nu(H_{jj} u_i^{n+1/2} + H_{ji} u_j^{n+1/2}) + f M u_i^{n+1/2} = 0 \tag{20}$$

$$M\,\frac{\eta^{n+1} - \eta^n}{\Delta t} + \bar{u}_i S_i h^{n+1/2} + \bar{u}_i S_i \eta^{n+1/2} + \overline{(h+\eta)} S_i u_i^{n+1/2} = 0 \tag{21}$$

$$M\,\frac{c_i^{n+1} - c^n}{\Delta t} + \bar{u}_i S_i c^{n+1/2} + \kappa H_{ii} c^{n+1/2} = 0 \tag{22}$$

where $u_i^{n+1/2}, h^{n+1/2}, \eta^{n+1/2}, c^{n+1/2}, \bar{u}_i$ and $\overline{h+\eta}$ are expressed as

$$u_i^{n+1/2} = \tfrac{1}{2}(u_i^{n+1} + u_i^n), \quad h^{n+1/2} = \tfrac{1}{2}(h^{n+1} + h^n) = h, \quad \eta^{n+1/2} = \tfrac{1}{2}(\eta^{n+1} + \eta^n)$$

$$c^{n+1/2} = \tfrac{1}{2}(c^{n+1} + c^n), \quad \bar{u}_i = \tfrac{1}{2}(3u_i^n - u_i^{n-1}),$$

$$\overline{h+\eta} = \tfrac{1}{2}(3(h+\eta)^n - (h+\eta)^{n-1}) = h + \tfrac{1}{2}(3\eta^n - \eta^{n-1})$$

where $\bar{u}_i$ and $\overline{(h+\eta)}$ are quasi-linear approximation of advection velocity, which is given by the Adams–Bashforth formula and has second order accuracy. Therefore, in the temporal direction, this discretization is the linear scheme which has second order accuracy.

3. OPTIMAL CONTROL THEORY

3.1. Performance function

The optimal control theory is employed to obtain the control value for the objective state value. For the control value, the angle of rotation of the bank located in the shallow water flow is employed. The performance function expressed by the square form of pollutant concentration is used. The performance function is expressed as

$$J = \frac{1}{2}\int_{t_0}^{t_f}\int_{\Omega}(c - c_{\mathrm{opt}})^{\mathrm{T}}Q(c - c_{\mathrm{opt}})\,\mathrm{d}\Omega\,\mathrm{d}t \tag{23}$$

where Q is weighting constants, c is the computed pollutant concentration and c_{opt} is the required concentration. The purpose is to find the optimal angle of rotation of the bank so as to minimize the performance function, which means the computed concentration should be as close as the optimal concentration. Here, the optimum concentration is the preassigned value, which is considered as the optimal state of the environment.

3.2. Derivation of adjoint equation

The performance function is constrained by the state equation. The Lagrange multiplier method is suitable for the minimization problem of the performance function with a constraint condition. Therefore the extended performance function is introduced using the Lagrange multiplier multiplied by the state equation. As the state equation which is added to the performance function, the discretized finite element equations in the spatial direction are employed. The extended performance function is expressed as

$$\begin{aligned} J^* = J &+ \int_{t_0}^{t_f}\int_{\Omega} u_i^{*\mathrm{T}}(M\dot{u}_i + \bar{u}_j S_j u_i + gS_i\eta + v(H_{jj}u_i + H_{ji}u_j) + fMu_i)\,\mathrm{d}\Omega\,\mathrm{d}t \\ &+ \int_{t_0}^{t_f}\int_{\Omega} \eta^{*\mathrm{T}}(M\dot{\eta} + \bar{u}_i S_i h + \bar{u}_i S_i \eta + \overline{(h+\eta)}S_i u_j)\,\mathrm{d}\Omega\,\mathrm{d}t \\ &+ \int_{t_0}^{t_f}\int_{\Omega} c^{*\mathrm{T}}(M\dot{c} + \bar{u}_i S_i c + \kappa H_{ii} c)\,\mathrm{d}\Omega\,\mathrm{d}t \end{aligned} \tag{24}$$

where u_i^*, η^*, c^* denote Lagrange multipliers for water velocity, water elevation and pollutant concentration, respectively. Matrices $M, S_i, S_j, H_{ii}, H_{ji}$ and H_{jj} denote the coefficients derived by the finite element equations. The extended performance function is divided into two parts which are referred to as the Hamiltonian and the time derivative term as follows:

$$J^* = \int_{t_0}^{t_f}\int_{\Omega}(H - u_i^{*\mathrm{T}}M\dot{u}_i - \eta^{*\mathrm{T}}M\dot{\eta} - c^{*\mathrm{T}}M\dot{c})\,\mathrm{d}\Omega\,\mathrm{d}t \tag{25}$$

where the Hamiltonian is expressed as

$$\begin{aligned} H = {}& \tfrac{1}{2}(c - c_{\mathrm{opt}})^{\mathrm{T}}Q(c - c_{\mathrm{opt}}) \\ &+ u_i^{*\mathrm{T}}(-\bar{u}_j S_j u_i - gS_i\eta - v(H_{jj}u_i + H_{ji}u_j) - fMu_i) \end{aligned}$$

$$+\eta^{*\mathrm{T}}(-\bar{u}_i S_i h - \bar{u}_i S_i \eta - \overline{(h+\eta)} S_i u_i)$$
$$+c^{*\mathrm{T}}(-\bar{u}_i S_i c - \kappa H_{ii} c) \tag{26}$$

As a necessary condition by which J^* should be stationary, the first variation of J^* must be zero. Therefore the terminal condition can be obtained as

$$u_i^* = 0, \quad \eta^* = 0, \quad c^* = 0 \quad \text{at } t = t_f \tag{27}$$

Calculating the gradient of Hamiltonian with respect to the state variable, the adjoint equations can be obtained in the following form:

$$M\dot{u}_i^* = -\frac{\partial H}{\partial u_i} \tag{28}$$

$$M\dot{\eta}^* = -\frac{\partial H}{\partial \eta} \tag{29}$$

$$M\dot{c}^* = -\frac{\partial H}{\partial c} \tag{30}$$

3.3. Finite element discretization for adjoint equation

Differentiating the Hamiltonian with respect to the state variable, the adjoint finite element equations discretized in the space direction can be obtained as

$$M\dot{u}_i^* - \Big[\bar{u}_j S_{,j}^{\mathrm{T}} u_i^* + \tfrac{1}{4} e\{S_{,i} u_j\}^{\mathrm{T}} u_j^* + \nu(H_{,jj}^{\mathrm{T}} u_i^* + H_{,ij}^{\mathrm{T}} u_j^*) + f M^{\mathrm{T}} u_i^*$$
$$+ \tfrac{1}{4} e\{S_{,i} h\}^{\mathrm{T}} \eta^* + \tfrac{1}{4} e\{S_{,i} \eta\}^{\mathrm{T}} \eta^* + \bar{h} S_{,i}^{\mathrm{T}} \eta^* + \bar{\eta} S_{,i}^{\mathrm{T}} \eta^* + \tfrac{1}{4} e\{S_{,i} c\}^{\mathrm{T}} c^* \Big] = 0 \tag{31}$$

$$M\dot{\eta}^* - \big[g S_i^{\mathrm{T}} u_i^* + \bar{u}_i S_i^{\mathrm{T}} \eta^* + \tfrac{1}{4} e\{S_i u_i\}^{\mathrm{T}} \eta^*\big] = 0 \tag{32}$$

$$M\dot{c}^* - [\bar{u}_i S_{,i}^{\mathrm{T}} c^* + \kappa H_{,ii}^{\mathrm{T}} c^*] + Q(c - c_{\mathrm{opt}}) = 0 \tag{33}$$

where e means the unit vector.

For the temporal discretization of the adjoint finite element equations, the Crank–Nicolson method is applied. Therefore, the adjoint finite element equations discretized in the temporal direction can be obtained as

$$M\,\frac{u_i^{*n} - u_i^{*n-1}}{\Delta t} - \Big[\bar{u}_j S_{,j}^{\mathrm{T}} u_i^{*n-1/2} + \tfrac{1}{4} e\{S_{,i} u_j\}^{\mathrm{T}} u_j^{*n-1/2} + \nu(H_{,jj}^{\mathrm{T}} u_i^{*n-1/2} + H_{,ij}^{\mathrm{T}} u_j^{*n-1/2})$$
$$+ f M^{\mathrm{T}} u_i^{*n-1/2} + \tfrac{1}{4} e\{S_{,i} h\}^{\mathrm{T}} \eta^{*n-1/2} + \tfrac{1}{4} e\{S_{,i} \eta\}^{\mathrm{T}} \eta^{*n-1/2} + \bar{h} S_{,i}^{\mathrm{T}} \eta^{*n-1/2} + \bar{\eta} S_{,i}^{\mathrm{T}} \eta^{*n-1/2}$$
$$\times \tfrac{1}{4} e\{S_{,i} c\}^{\mathrm{T}} c^{*n-1/2} \Big] = 0 \tag{34}$$

$$M\,\frac{\eta^{*n}-\eta^{*n-1}}{\Delta t}-\left[gS_i^{\mathrm{T}}u_i^{*n-1/2}+\bar{u}_iS_i^{\mathrm{T}}\eta^{*n-1/2}+\tfrac{1}{4}e\{S_iu_i\}^{\mathrm{T}}\eta^{*n-1/2}\right]=0 \tag{35}$$

$$M\,\frac{c^{*n}-c^{*n-1}}{\Delta t}-[\bar{u}_iS_{,i}^{\mathrm{T}}c^{*n-1/2}+\kappa H_{,ii}^{\mathrm{T}}c^{*n-1/2}]+Q(c-c_{\mathrm{opt}})=0 \tag{36}$$

where $u_i^{*n-1/2}$, $\eta^{*n-1/2}$ and $c^{*n-1/2}$ are expressed by

$$u_i^{*n-1/2}=\tfrac{1}{2}(u_i^{*n}+u_i^{*n-1}),\quad \eta^{*n-1/2}=\tfrac{1}{2}(\eta^{*n}+\eta^{*n-1}),\quad c^{*n-1/2}=\tfrac{1}{2}(c^{*n}+c^{*n-1})$$

4. TIME DOMAIN DECOMPOSITION METHOD

In principle, the solutions of the state equation at all discretization points in space and in time are required in order to solve the adjoint equation. For large scale optimization problem, lot of computational storage space is required and it is almost impossible to store the solution of the state equation at every time discretization point. Thus, the method, which can drastically reduce the storage requirements is described below. The time domain decomposition method is employed as a technique for reducing storage requirements [5, 6]. The algorithm of the time domain decomposition method can be described as follows:

1. Assume the number of time steps N.
2. Consider the positive integers A and B, the meanings of A and B are the number of division and the number of time steps in the divided section, so that $AB=N$.
3. Decompose the interval $[0,t_f]$ in A subinterval of length $B\Delta t$.
4. Solve the state equation and store the solution for $n=iB$ and $i=0,1,\ldots,A-1$, and then for $n=(A-1)B+j$ and $j=1,2,\ldots,B$, which are shown in Figure 2.
5. Solve the adjoint equation for $n=N,N-1,\ldots,N-B+1$ using the solution of state equation for $n=N-B+1,N-B+2,\ldots,N$ and compute the gradient of performance function.
6. Solve the state equation and store the solution for $n=(A-2)B+1,(A-2)B+2,\ldots,(A-1)B-1$.
7. Solve the adjoint equation for $n=(A-1)B-1,(A-1)B-2,\ldots,(A-2)B+1$ using the solution of the state equation for $n=(A-2)B+1,(A-2)B+2,\ldots,(A-1)B-1$ and compute the gradient of performance function.
8. Set $A=A-1$ and go to step 6.

The number of storage requirements is $A+B$. Thus, to minimize the storage, $A+B$ should be minimized. If $N^{1/2}$ is an integer, the minimum is attained for $A=B=N^{1/2}$. Therefore the

Figure 2. Subdivision of $[0,t_f]$, $A=5$, $B=5$, $N=25$.

storage requirements is $2N^{1/2}$ instead of N. For the large scale optimization problem, this method is efficient for the computation.

5. MINIMIZATION TECHNIQUE

5.1. Sakawa–Shindo method

The Sakawa–Shindo method is applied for the minimization technique [7]. In this method, the modified performance function to which a penalty term is added is introduced. The modified performance function is written as

$$K = J^{*(l)} + \frac{1}{2}(\bar{\theta}^{(l+1)} - \bar{\theta}^{(l)})^{\mathrm{T}} W^{(l)} (\bar{\theta}^{(l+1)} - \bar{\theta}^{(l)}) \tag{37}$$

where l is the iteration number for the minimization, $\bar{\theta}$ is the angle of rotation and $W^{(l)}$ is the weighting parameter. The angle of rotation expresses the locational direction of the bank to prevent the dispersion of pollutant. Applying the stationary condition, $\delta K = 0$, the following equation can be obtained. The angle of rotation of the bank is renewed by the following equation:

$$W^{(l)}\bar{\theta}^{(l+1)} = W^{(l)}\bar{\theta}^{(l)} + \int_{t_0}^{t_f} \int_{\omega} \frac{\partial H}{\partial \bar{\theta}}\, \mathrm{d}\omega\, \mathrm{d}t \tag{38}$$

where ω means the domain of rotation and the gradient of the Hamiltonian with respect to the angle of rotation is expressed as

$$\begin{aligned} \frac{\partial H}{\partial \bar{\theta}} = {} & u_i^{*\mathrm{T}} \left(-\bar{u}_j \frac{\partial S_j}{\partial \bar{\theta}} u_i - g \frac{\partial S_i}{\partial \bar{\theta}} \eta - v \left(\frac{\partial H_{jj}}{\partial \bar{\theta}} u_j + \frac{\partial H_{ji}}{\partial \bar{\theta}} u_j \right) - f \frac{\partial M}{\partial \bar{\theta}} u_i \right) \\ & + \eta^{*\mathrm{T}} \left(-\bar{u}_i \frac{\partial S_i}{\partial \bar{\theta}} h - \bar{u}_i \frac{\partial S_i}{\partial \bar{\theta}} \eta - \overline{(h+\eta)} \frac{\partial S_i}{\partial \bar{\theta}} u_i \right) \\ & + c^{*\mathrm{T}} \left(-\bar{u}_i \frac{\partial S_i}{\partial \bar{\theta}} c - \kappa \frac{\partial H_{ii}}{\partial \bar{\theta}} c \right) \end{aligned} \tag{39}$$

The algorithm of the Sakawa–Shindo method is shown as follows:

1. Choose an initial control value $\bar{\theta}^{(l)}$.
2. Compute $u_i^{(l)}$, $\eta^{(l)}$ and $c^{(l)}$ by the state equation and the initial performance function $J^{(l)}$.
3. Compute $u_i^{*(l)}$, $\eta^{*(l)}$ and $c^{*(l)}$ by the adjoint equation.
4. General a new angle of rotation $\bar{\theta}^{(l+1)}$.
5. Check for convergence; if $\|\bar{\theta}^{(l+1)} - \bar{\theta}^{(l)}\| < \varepsilon$, then stop, else go to step 6.
6. Compute $u_i^{(l+1)}$, $\eta^{(l+1)}$ and $c^{(l+1)}$ by the state equation and the performance function $J^{(l+1)}$.
7. Renew a weighting parameter $W^{(l)}$; if $J^{(l+1)} \leqslant J^{(l)}$, then set $W^{(l+1)} = 0.9W^{(l)}$ and go to step 3.
   else set $W^{(l+1)} = 2.0W^{(l)}$ and go to step 4.

6. SHEAR-SLIP MESH UPDATE METHOD

For the rotational problem of the body, the shear-slip mesh update method is usefully adaptable [8]. In the case that the body is rotated, remeshing is required. As one of the remeshing techniques, this method is useful. The details of the shear-slip mesh update method are illustrated in Figure 3. The whole domain is divided into three regions which are the domain of rotation, the connection domain and the fixed domain. When the body is rotated, the mesh data are deformed in the connection domain. This method is based on the fact that if the mesh data are deformed, the only mesh data in the connection domain are updated. Figure 4 shows the way to update the mesh data. This method is very efficient for the problem of rotational body. In this research, this method is used to renew the angle of rotation of the bank.

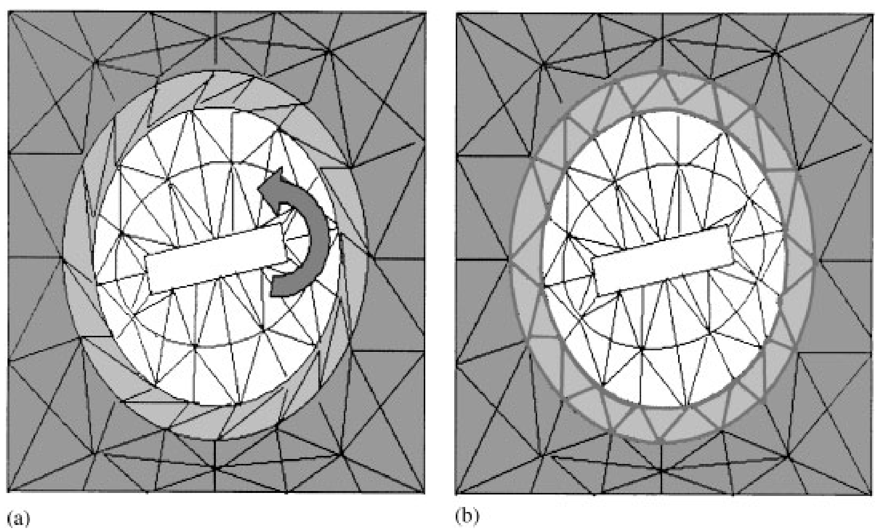


Figure 3. Shear-slip mesh update method: (a) before application; (b) after application.

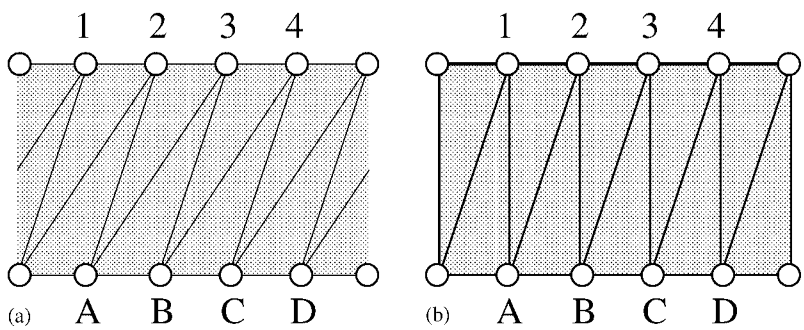


Figure 4. Renewal of mesh data: (a) before application; (b) after application.

7. CASE STUDIES

7.1. Validation using rectangular channel

The rectangular channel is employed to validate the method presented in this paper. The purpose of this research is to investigate the effect of the location of the bank and to obtain the optimal angle of rotation of the bank using the optimal control theory. The finite element mesh and the computational condition are illustrated in Figures 5 and 6, respectively. The Kalman constant κ_l is 0.41, the Manning coefficient of roughness is 0.001 (sec/m$^{1/3}$) and the diffusion coefficient κ is 0.5 (m^2/s). Total number of nodes and elements are 1121 and 2094, respectively. The pollutant concentration, 10 (ppm) is made to flow from the central point on the left side boundary of the rectangular channel. Time increment Δt is 0.05 (s), the number

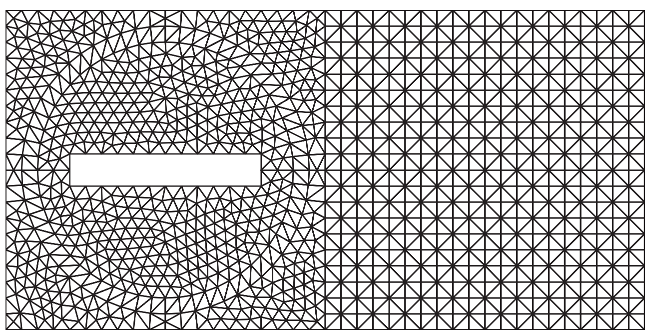


Figure 5. Finite element mesh.

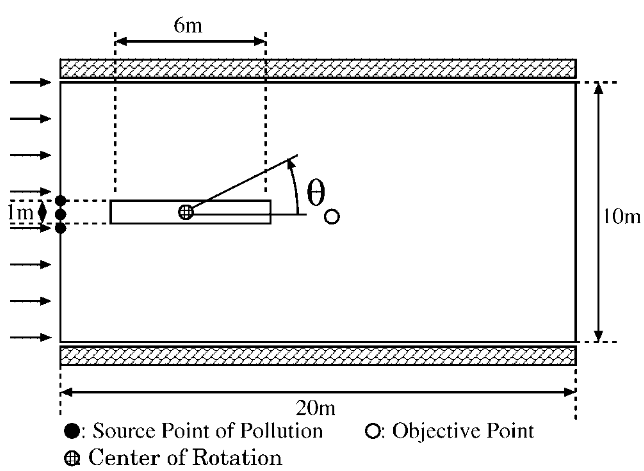


Figure 6. Computational condition.

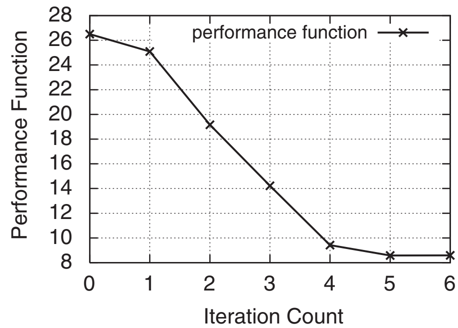


Figure 7. Variation of performance function.

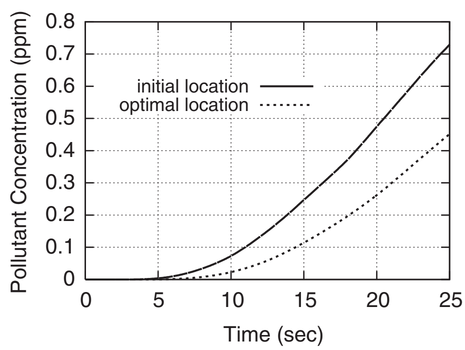


Figure 8. Time history of pollutant concentration at the objective point.

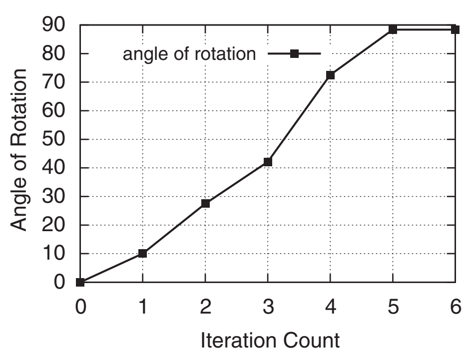


Figure 9. Variation of angle of rotation.

of time steps is 1000, the number of divisions A is set as 25, the number of time steps B in the divided section is set as 40. The weighting constant Q at the objective point is 1.00 and at other points is 0.00.

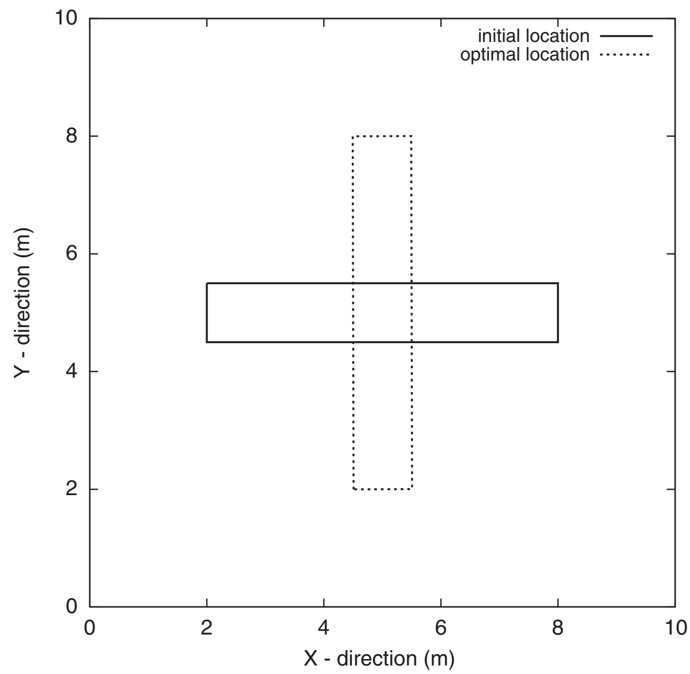


Figure 10. Comparison of the location of the bank.

Table I. Reduction of computational storage requirements.

	EXE size
Conventional method	208.9 (MB)
TDDM	126.0 (MB)

Figures 7 and 8 show the variation of the performance function and the time history of pollutant concentration at the objective point. The performance function decreased monotonically and converged. In the final solution, it is natural that the barrier is perpendicular to the line from the source to the objective points. This means, the validation of the present method has been confirmed. The fact that the pollutant concentration is reduced can be confirmed. But it is also found that there exists a residual pollutant. Therefore, whether the pollution can be reduced or not depends on the diffusion coefficient. Figures 9 and 10 show the variation of the angle of rotation and a comparison of the location of the bank. The computed angle of rotation is 88.35 degrees.

Table I shows that the computational storage requirements are reduced from 208.9 (MB) to 126.0 (MB) by using the time domain decomposition method. The computational storage requirements could be further reduced to 39.7%.

7.2. Onjuku coast

Onjuku coast located in Chiba prefecture in Japan is analysed for practical computation. The coast is polluted by the effluents flowing into it from the Shimizu river. The location of Onjuku coast is shown in Figure 11. The pollution is accumulated in the coast off from the Iwawada port. Therefore, it is reported that the ear shells could not be caught as before. The fishing point is off the coast of the Iwawada port, which is indicated in Figure 13.

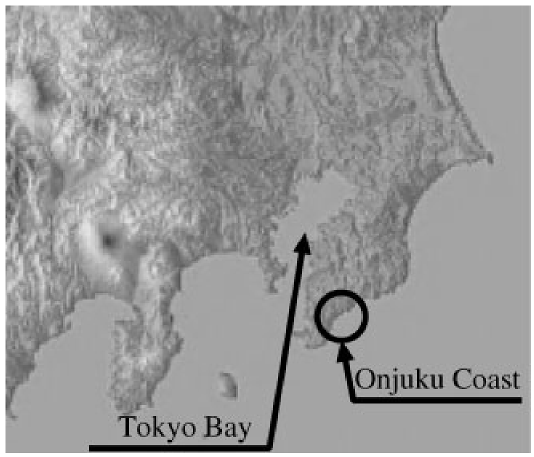


Figure 11. Location of Onjuku coast.

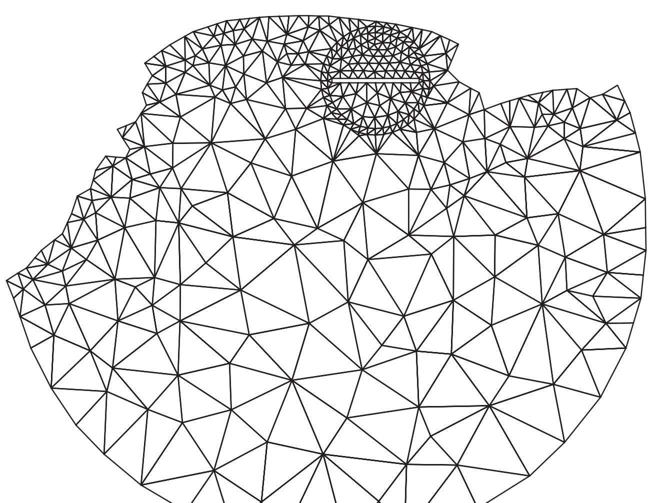


Figure 12. Finite element mesh.

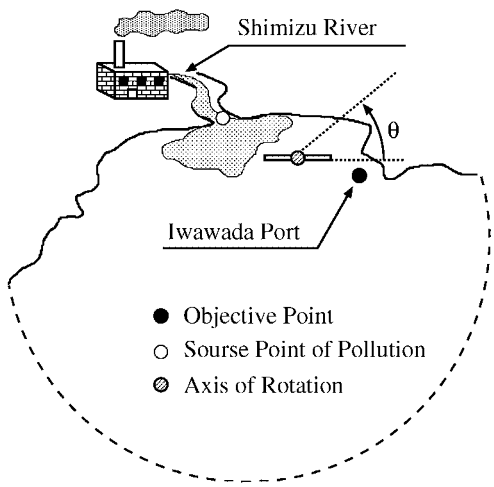


Figure 13. Computational condition.

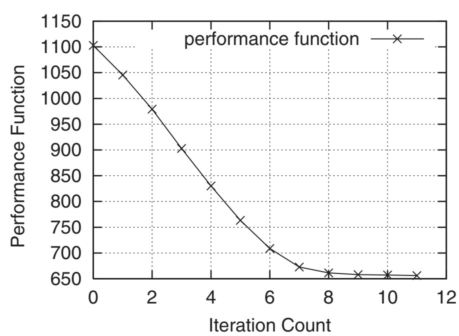


Figure 14. Variation of performance function.

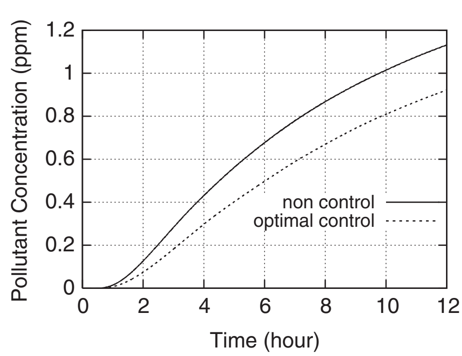


Figure 15. Time history of pollutant concentration at the objective point.

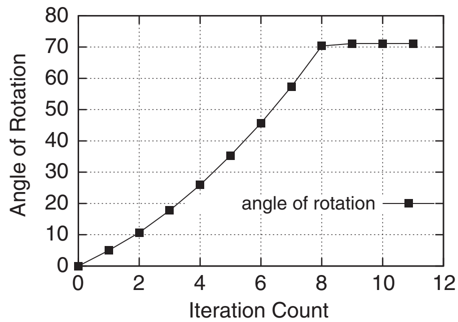


Figure 16. Variation of angle of rotation.

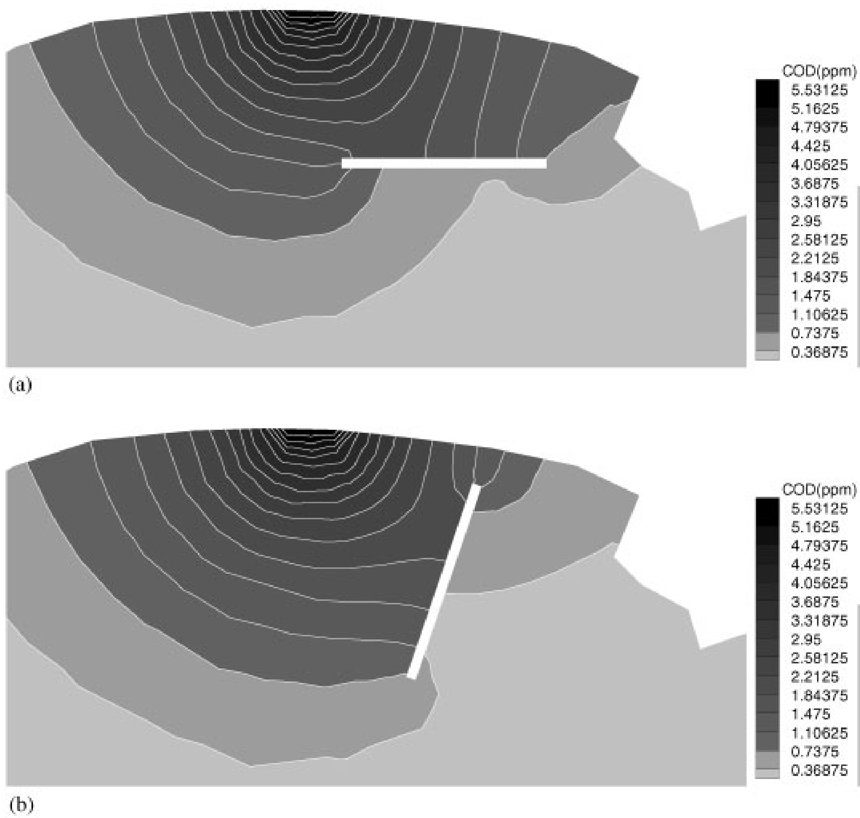
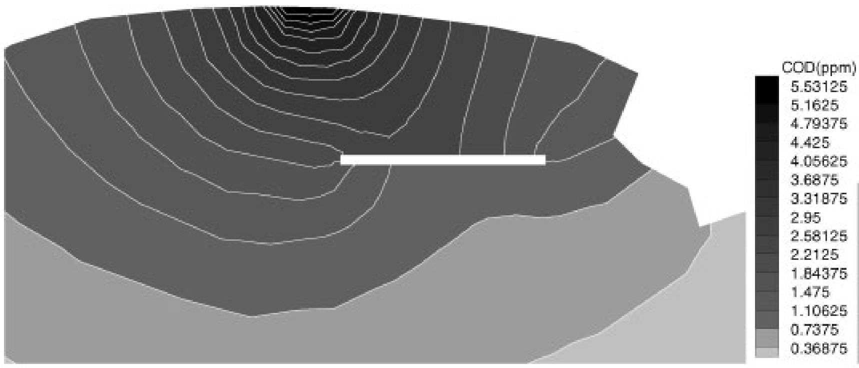
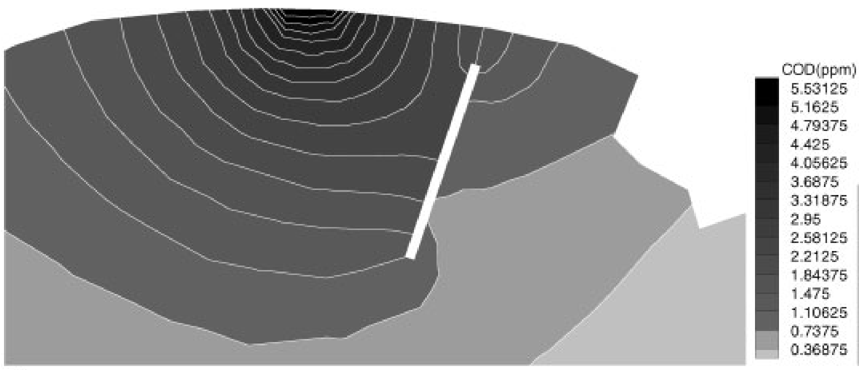


Figure 17. Non-control case and optimal control case for pollutant concentration each 3 h. (a) Non-control case (after 3 h); (b) optimal control case (after 3 h); (c) non-control case (after 6 h); (d) optimal control case (after 6 h); (e) non-control case (after 9 h); (f) optimal control case (after 9 h); (g) non-control case (after 12 h); and (h) optimal control case (after 12 h).



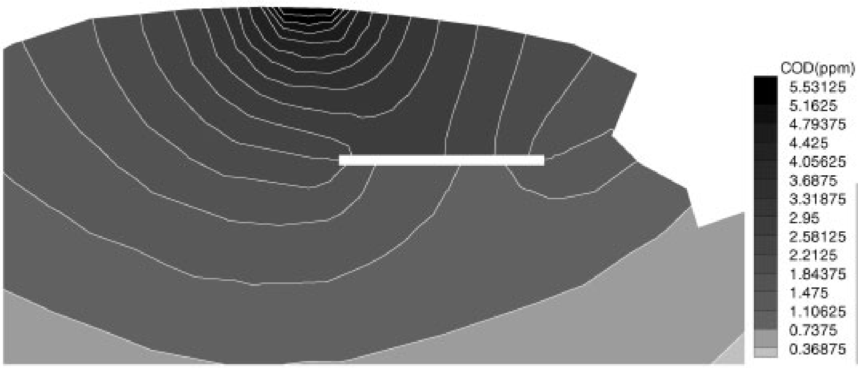
(c)



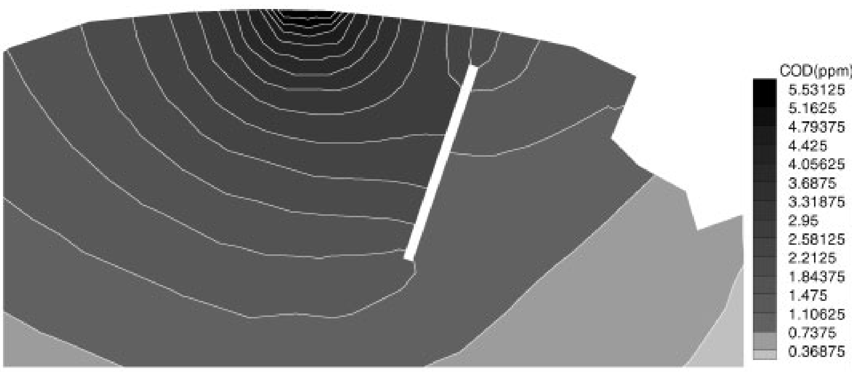
(d)

Figure 17. (*continued*).

To settle this problem, the control method of pollution by a bank is suggested. The bank for controlling the pollution is set between the Shimizu river and the Iwawada port. The pollutant concentration is accumulated easily in the area off the coast of the Iwawada port by the advection effect of the local flow in this area. The purpose of this research is to investigate the effect of this plan and to obtain the optimal angle of rotation of the bank using the optimal control theory. The finite element mesh and the computational condition are illustrated in Figures 11 and 12, respectively. Total number of nodes and elements are 434 and 745, respectively. The pollutant concentration, 5.9 (ppm), is made to flow from the mouth of the Shimizu river. From the open boundary, the water velocities and the water elevation obtained by the Kalman filter finite element method are given [9]. The Kalman filter finite element method is one of the estimation methods of tidal flow using the limited observation data. Thus, the boundary condition of open boundary is estimated by the observation data, and the flow behaviour is analysed by the estimated tidal boundary condition. Time increment Δt is 10.0 (s), the number of time steps is 4320, the number of divisions A is set as 60, the number of time steps B in the divided section is set as 72, the Kalman constant κ_l is 0.41, the Manning coefficient of roughness is 0.03 (s/m$^{1/3}$) and the diffusion coefficient κ



(e)



(f)

Figure 17. (*continued*).

is 10.0 (m^2/s). The weighting constant Q at the objective point is 1.00 and at other points is 0.00.

Figures 13, 14 and 15 show the computational condition, the variation of the performance function and the time history of the pollutant concentration at the objective point, respectively. The performance function was monotonically decreased and converged, which means the pollutant concentration is controlled. But, in this computation, it is found that the reduction of pollutant concentration is not satisfactory. Figure 16 shows the variation of the angle of rotation. The computed angle is 71.14 degrees. It is considered that the computed angle of the rotation of the bank becomes perpendicular to the connected line between the source point of pollution and the objective point. Figure 17 shows the distribution of the pollutant concentration near the Iwawada port in both the non-control and the controlled cases. In the controlled case, the reduction of pollutant concentration can be confirmed. The pollutant concentration at the objective point could be reduced 18.5% at the terminal time.

Table II shows that the computational storage requirements are reduced from 431.6 (MB) to 112.8 (MB) by using the time domain decomposition method. The computational storage requirements could be further reduced to 73.9%.

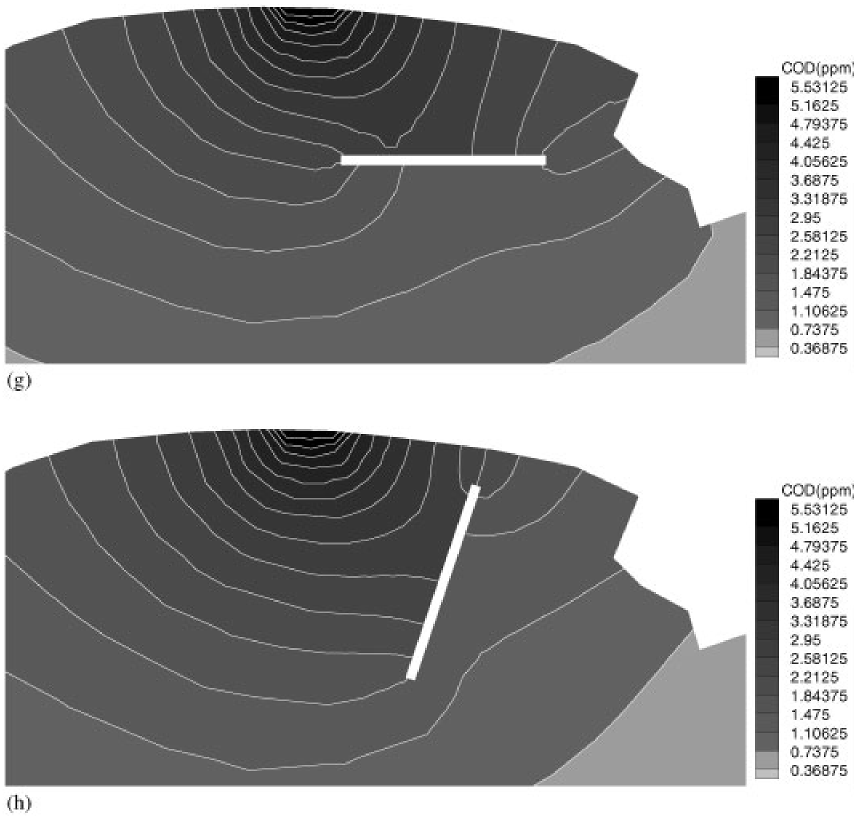


Figure 17. (*continued*).

Table II. Reduction of computational storage requirements.

	EXE size
Conventional method	431.6 (MB)
TDDM	112.8 (MB)

8. CONCLUSION

In this paper, the optimal control theory was presented to decide the optimal angle of rotation of the bank for controlling the pollutant concentration at the objective point. As the state equations, the shallow water equation and the advection-diffusion equation are employed. The Galerkin method with bubble function element and Crank–Nicolson method are used for spatial discretization and temporal discretization. As a computational storage requirements reduction technique, the time domain decomposition method is applied to the optimal control problem. The computational storage requirements could be drastically reduced. Especially, for the long time and large scale optimization problem, it is considered that this method is shown to be very efficient. The Sakawa–Shindo method is applied to the minimization technique. The shear-slip mesh update method is used for the rotation of the bank.

The optimal angle of rotation of the bank could be obtained using the optimal control theory. It is considered that the obtained location of the bank is suitable for the prevention of diffusion of pollutant concentration. The reduction of pollutant concentration by the bank could be confirmed. For future works, it is necessary to apply the multi-objective points and multi-bank problems. Moreover, it is necessary to decide the appropriate terminal time.

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